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by

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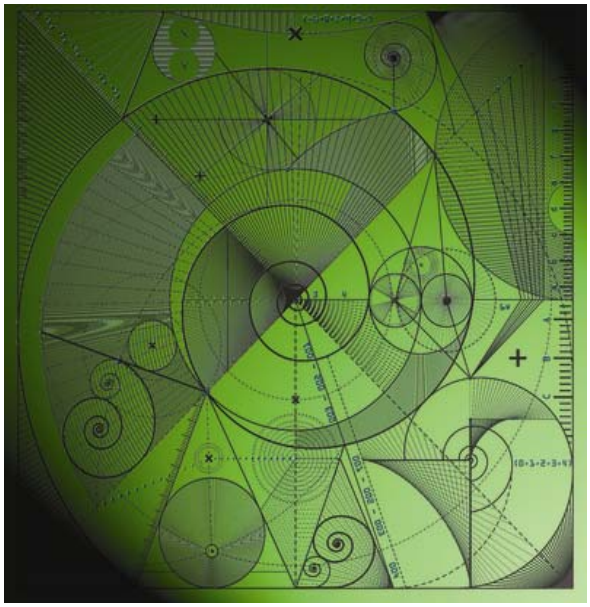

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1 Introduction

There has never been a time in history when the world has been more connected. It is a condition that is going to increase. From the light speeds of the internet and television communication, passing content and material between countries that are increasingly tightly coupled; to the growth of emails, and citizen journalism, exploiting video phones and Twitter; to the empowerment of individuals through blogs and social media. Events in one location can have a major impact on the audiences in other parts of the world; hardening attitudes and creating polarised reactions. Now we have a "Digital Society" [1] within which everybody is both close up to and far away from everybody else. The *Vietnam War effect*, where attitudes about the military involvement and its raison d'etre were shaped in the living rooms of America, is now a business that operates on a 24-7 basis. Opinions, beliefs and attitudes around the world are being modulated by the images conveyed by the media and by individuals. It is in an emotional environment (such as where the scenes transmitted around the world in, say, the direct aftermath of a drone attack that has killed apparently innocent civilians,) that radicalisation can occur. People, around the world, can suddenly feel it is their time to act and move beyond the passive to the active.

Ted Gurr's book [2] provides a range of insights as to why people become active and turn to violence. The Vietnam War protests provided the backdrop and demonstrated the power of the media to motivate people to become involved in political violence. The situation in the run up to the Second Gulf War showed these three factors writ large and operating at a fundamentally different level than seen during the Vietnam War.

Forty years later technology has provided a much greater range of ways in which people can become involved in social movements. The mobilisation structures of chat rooms, email, and the plethora of social networking sites

available today play a hugely important part in creating the conditions for people to become involved in some form of political violence. The media can be instrumental in facilitating the mobilisation structures. These structures, or channels, provide the means by which people can have their attitudes and beliefs shaped; seeing events through the magnifying glass of immediate perception.

In recent times the decision taken by the UK Government to go to war in Iraq was a catalyst for many vulnerable Muslims to act, who felt compelled to become involved either directly or indirectly in challenging that decision. The central message of Al Qaeda, that the west was seeking to destroy Islam, chimed with many in the United Kingdom who saw the daily carnage served up from the streets of Iraq. The attacks on the London Underground on the 7th of July 2005 provided a vivid reminder of the first and most violent form of political violence in reaction to the mobilising effect of the media.

At the start of the 21st century the so-called 'butterfly effect' is thus alive and well, as events in one part of the world are seen through by an international audience whose beliefs and attitudes are becoming increasingly stratified. It is through this clever manipulation of the media (both broadcast media and social media) that contemporary terrorist movements, such as Al Qaeda, maintain their presence. And may do so in the face of an onslaught from states collaborating in the introduction of legal frameworks and military activities that are designed to restrict their freedom of manoeuvre, through conducting what are called upstream operations.

Paradoxically, the impact of this mass media coverage has been to create a greater definition and distinctions of social identities, allied with an emergent sense of grievance fuelled by a variety of factors. The visions of those that invented Esperanto as a means by which a global identity in language would facilitate a possible move to a global collective set of values and beliefs has not materialised. The Nobel laureate V.S. Naipaul's imaginative vision of a universal civilisation has yet to appear. Samuel Huntingdons contrasting vision of a world fragmented by religious beliefs into distinct groupings that are in essence at various stages of confrontation with each other seems, superficially at least, to be a prescient analysis and highly contemporary.

In this fragmented and often highly charged atmosphere it is understandable that there are those that turn to violence to express their grievances and to try and achieve fundamental political change. Terrorism has seen a number of stages of development and numerous authors have offered views chronicling its development from the 19th century to its modern day inceptions. It is possible to dissect this period and to suggest several stages in the evolution of terrorism by focusing upon the motivation and intent of the groups involved. Many of these groups had specific political agendas, such as the Basque separatist movement Eta. This genre of terrorist groups were not particularly agile in the targets they selected or in the ways their used the media. That said the images of the Munich Olympics retain their ability

to shock. Munich was an example of an extreme event, one that stands out in the mind, masking a range of other targeted assassinations conducted by groups from the Red Brigades and Barder Meinhoff that at the time seemed quite extraordinary but since then have faded into relative obscurity.

These were the nascent stages of contemporary terrorism. While they enjoyed limited media coverage their impact on the global stage was contained to those communities in whose name the violence was conducted, such as the Palestinians. In western societies, that had only just started their journey to multi-culturalism, these acts of terrorism challenged fundamental attitudes and beliefs of the majority of their citizens. The reaction of the majority of the population was predictable. Terrorism at that time did not gain a great deal of traction with wider target audiences other than those who for a variety of reasons where politically active and whose agendas were to fundamentally challenge the structure and fairness of western society and its values. What is clear is that while it is generally accepted that terrorism had its beginnings in Russia its protean nature since those early days have created a wealth of challenges for scholarly analysis.

A clear and concise definition of terrorism is proving difficult to express as academics and those involved in law enforcement security wrestle with the subtle changes that occur on the boundaries between terrorism and the kind of insurgent movements we have recently witnessed in operation in Iraq, Afghanistan, Somalia and spreading out across Africa, driven by specific and narrow interpretation of Islam. The ideology of Al Qaeda is both its strength and its weakness. Military analysts would recognise its importance as a centre of gravity - a place from which Al Qaeda draws its strength. Politicians in contrast struggle to find the words that express the weaknesses in the ideology - being unable to challenge its fundamental tenants - and wary of creating a backlash as a result of unintended consequences.

Al Qaeda has proven adept at using the developments of the Internet to spread its message of Jihad and its core narrative that the west is seeking to destroy Islam. This narrative is important as it chimes with many people across the world who feel aggrieved and who lack a clear sense of their social identity. These are vulnerable people who can rapidly relate to the notion that from the evidence they see clearly on a daily basis through the media that Al Qaedas charge against the west is true.

Since coming to office President Obama has tried to develop a new strategic dialogue with the Muslim world. Initially, in the wave of hope that followed his election, this gained some traction and his visit to the centre of Muslim learning in Cairo was important. His speeches have sought to defuse the tensions between the west and the Muslim world. However the drum beat of civilian casualties in Afghanistan and the continued security uncertainties in Iraq and Pakistan have eroded much of that initial goodwill.

Throughout this period since the devastating attacks in the United States on September 11th 2001 Al Qaeda has proven to be a robust and resilient

organisation. The development of a series of regional franchises in the Yemen (Al Qaeda in the Arabian Peninsula) and in the Maghreb and the Sahel have shown how terrorist groups can fragment and maintain their relevance on the international stage. It has shown how easy it is for terrorist groups to morph and adapt to circumstances, paradoxically using the connectivity of the Internet and the media to reach out and affect public opinion; using events to maintain and heighten the sense of grievance felt by many in the Muslim world.

While it is agile, an organisation Al Qaeda does have its weaknesses. One of these is its inability to harness and focus the effort of the franchises. The groups involved, such as Al-Shabab in Somalia, have tended to focus upon their desire to build an Islamic state in Somalia. They focus on the near-enemy: the Transitional Government of Somalia. Recently in Somalia, Pakistan and the Yemen there is prima facia evidence emerging from a combination of actions and statements that indicate these organisations are moving from being locally focused onto a broader view of where they should conduct violence to achieve their ends. The failed Christmas Day attack over Detroit, the attack on the Japanese Oil Tanker in the Persian Gulf in August 2010 and the threat by AQAP and Al-Shabab in July to close the strategically vital Bab al-Mandab Strait at the southern end of the Red Sea are all measures that have an increasingly international agenda.

The current genre of terrorist groups show flexibility and agility in other ways, reaching out specifically to their target audiences and providing access to social services and humanitarian relief when natural disasters strike. One example of this agility that is inherent in contemporary terrorist movements has shown itself in Pakistan in the wake of the floods that affected much of the North East of the country in August 2010. Groups that the west would readily designate as terrorists moved quickly to fill the vacuum created by the absence of formal governance structures in the region controlled by Islamabad. This ability to deliver humanitarian aid into these situations is a tactic employed by groups, such as Hamas and Hezbollah. Where state government fails, non-state actors show great agility and opportunism in seizing moments where the state-based government fails to enhance their standing with people in the local community. Their support in such difficult times is partly politically motivated. But it also has an underlying and more sinister intent; to secure the supply of foot-soldiers that can maintain their campaign of violence. Contemporary terrorist movements know how to pull a range of levers to achieve their aims.

The terrorist attacks in Mumbai were designed to have an impact on a global stage. They were to have a strategic impact; with news coverage lasting for days as the Indian authorities struggled to contain the situation. Given the way the events played out through the media it is not difficult to see why some commentators have labelled this the Indian equivalent of September 11th.

The attacks in Mumbai and the sectarian nature of the violence in Iraq in the wake of the invasion in 2003 have arguably created a new genre of terrorism that seeks mass casualty attacks even in places of worship. Pakistan has suffered a number of attacks that have specifically targeted Muslims whose interpretation of Islam differs as they follow other schools, such as Sufism. These tactics, aimed at specific audiences, belie the weaknesses at the heart of the ideology of Al Qaeda. They are vulnerable to accusations that they adopt a stance of selective reading of the Koran; placing a greater emphasis upon their literal interpretations of what are sometimes referred to as the Sword Verses.

One specific issue that goes to the heart of this selective analysis are the arguments that arise from the accusation that when Al Qaeda and its affiliates kill Muslims that this is not permitted in the Koran. Actions where such events occur routinely create a knock-on effect in social network sites; where debates rage between people with varying claims as to their legitimacy. Increasingly the justification at the heart of Al Qaedas ideology is being questioned. Learned Muslim scholars are feeling able to come forward and challenge the interpretations of the Holy Book. The theological study published in London by the leading Muslim cleric Dr Qadri is an example of this trend. Muslims are therefore being bombarded with a range of viewpoints and analysis of what is right and wrong; through a range of communications channels. The situation they face is a dynamic one as people with differing claims to legitimacy offer what might to some appear to be contradictory interpretations of the cannon of Islam.

It is against this complex backdrop that law enforcement and security authorities in the west must operate. Many of the issues discussed above fuel sentiment in western societies that are becoming increasingly multicultural. There is however an asymmetry at the heart of development of these new societies where the benefits of being a member of the community do not appear to be fairly distributed. For many, democracy is not a shining example of how to live their lives; the benefits accrued by others do not seem to flow fairly and racism remains an enduring problem.

This environment affects the way that many second generation immigrants into western societies view their lives. They become prone to having their traditional attitudes and beliefs instilled in them by their parents challenged. They find it increasingly difficult to reconcile the actions of their Government, such as the when the United Kingdom armed forces participated in the invasion of Iraq against widespread public disquiet, with their own emergent perspective. The seeds of these views were sown in the conflict in the Balkans when the west appeared to stand by and do nothing to help when Muslim populations were the victim of genocide. Russians wars in Chechnya also added fuel to fires and created a large repository of imagery and videos that still circulate on the Internet; infecting young and impressionable minds.

Articulating all of the factors that shape peoples minds and lead to some of them deciding to take action and resort to violence is difficult. This is what is known as a wicked problem where many parameters interact and first, second and third order effects can become intermingled. Decoupling this kind of tightly-coupled situation is difficult. What used to be a distinct layer of first, second and third order set of relationships can no longer be seen in a hierarchical form; in an emotionally charged atmosphere all effects are first order.

There is an increasing literature documenting this field. For new entrants (especially from the sciences), the following sources may provide helpful background [3, 4, 5, 6, 7, 8, 9, 10].

It is axiomatic that when trying to deal with complex situations that simple models facilitate understanding; abstraction helps. One way of looking at the problem of radicalisation is to represent the psychological and sociological factors at work in the form of the well known game of **snakes and ladders**.

Figure 1. Snakes and Ladders for a variety of individual journeys.

The aim of the terrorist movements is to create the conditions where people, who feel marginalised and on the periphery of society, who are unsure of their social identity, decide to look deeper into their heritage. They are looking for meaning and seek solace in their religion. In our representation of radicalisation they chose to pick up the dice and play the game of snakes and ladders. Ignoring the rule that you have to throw a six to start the game they move along the board, taking steps that are determined by the results of each throw of the dice. Occasionally they will land on a ladder. This rapidly advances their position on the board, a distinct step has been taken along the trajectory of being radicalised - narrowing their world view and hardening their attitudes.

Some may find it important to pause on the journey; seeking to reconcile dissonant feelings that may surround them on a daily basis as they try to rationalise the potentially violent end in which the journey may end. External events can have a dramatic effect upon these individuals. Things they witness through the media or the Internet can suddenly create tipping points; in effect creating unexpected ladders. Attitudes and beliefs become further narrowed and fervour for action is enhanced. At this point the potential for intervention to help the candidate leave the journey becomes increasingly difficult; especially if they are part of a small group or cell whose social identity is gradually forming into a group identity with all of its associated implications.

The further the candidate moves along the journey the narrower their views on the world become; groups follow a similar pathway sometimes led by a single articulate leader who through their personality takes the members on the journey with them. This narrowing of their lens on the world, in part, helps them address dissonant feelings as they learn to self-justify their actions enabling them to continue the journey. Ladders can not only arise in unexpected places but also vary in height. They are not all necessarily large ladders. Sometimes events can have less impact as the media quickly moves onto another subject. The media is notoriously fickle and its coverage can move on, but the fires its coverage has created burn brightly in the chat rooms and for those on the journey keep the candle alight for those reported to have been affected in the original media coverage.

While it is relatively easy to appreciate the impact of ladders providing a stimulus for sudden surges in emotions and feelings along the journey, the creation of ways out of this trajectory to violence are less straightforward to define. In our representation it is the snakes that provide the exit routes from the journey to violence. The task for the community at large, and its political leaders and local representatives, is to avoid creating ladders whilst at the same time helping create the conditions where snakes can be formed to take people off the board. Ideally we need the individuals not just to slip down the snake to resume the game, but to stop playing altogether.

Approaches to the task of creating snakes vary. In several countries in the Middle East, such as Saudi Arabia, there are rehabilitation programs that are provided by the government to help people turn away from terrorism. Recent evidence has shown that these programs are having a mixed effect, with some people appearing to benefit and others using it as a means to gain financing for terrorist groups; taking the monetary rewards for having attended and them leaving to travel to the Yemen.

Despite the mixed messages emerging for these attempts to, in our language, create snakes it is clear that a systematic effort needs to be made to more effectively communicate the views of those that are prepared to stand up to the extremists. It is possible to think of these measures as being taken at a strategic and operational level, alongside the attempts by President

Obama to adopt a more conciliatory rhetoric removing the language of the war on terror with all its ability to be manipulated by terrorist movements such as Al Qaeda. Other measures need to be developed and implemented at the operational [within state] and at the local level [within communities] that can help ensure that those vulnerable to being tempted into terrorism do not embark upon the journey or choose to leave it before they reach 100.

Our interests reflect some recent similar thinking by other authors, including those contributing to [12, 13], where there is interest in some alternative approaches within the mathematical modelling of distinct aspects of terrorism and counter terrorism.

In this paper we are suggesting that the value of the snakes and ladders concept it threefold.

Firstly it is a very useful framework to communicate the simple progression of individuals' journeys against complex and multifaceted issues or geography, society, race, culture, religion, ideology, communication and events. It is the progression and radicalisation of individuals that results in the threat from the most active and radical. Any snap shot shows a population distribution of individuals over the whole board, that may be slow to change: but it the the *Lagrangian* experience of the individuals moving through the population that is important - and the stochastic nature of individuals' own progressive steps, together with the imposition of the ladders and snakes, that results in a range of orbits - from square one to square one hundred. For individuals who stay the course the time taken to become radicalized and active may be as little as six weeks, for others it may take a median of nine to fifteen months.

Secondly the concept suggests a number of approaches that might help deal with the whole process. And mathematics may have something to say about these. For example monitoring and managing individuals known to have reached high squares is extremely expensive: so what effort should be expended lower down the board to reduce this population (how should snakes be located at what costs?); and what is the return on investment of such activity? Or what is the effect of internationalization? Having parallel but coupled boards each representing populations in different countries, we may see some dynamic effects such as destabilizing and otherwise set of isolated systems. We will discuss such example phenomena later. The point is that the framework allows us to pose questions and make use of methods and models developed for other applications of similar dynamical systems. The modelling, inference, forecasting and control ideas become easier and for transparently deployed. The task here is NOT to define an all feature model and subject it to a mathematical analysis, but rather to consider what different facets of mathematics might bring to the distinct issues raised by adopting the model framework. So we seek a collage of mathematical insight describing various features and sub issues.

Thirdly it is possible that the concept itself could drive future data col-

lection. While beyond the scope of the current paper, this exploitation is in our minds. If new (news) events (initially) radicalize a large population to join the board; how will we detect and monitor this? And how do we expect any possible interventions to prevent such a population bulge surviving up to the highest squares, and how soon should any interventions be timed? At this stage we can only speculate: but there could be useful methods of data collection informing not just the populations at various squares on the board, but also estimating the dynamic progression rates, from square to square, and via whatever ladders can be identified. The dynamics of the transitions need to inform the dynamics (timing and duration) of the response. Moreover perhaps we could infer the existence of ladders from aberrant journeys (narratives)?

Making the snakes and ladders concept become a quantitive tool is certainly a possibility - but here we focus on the conceptual and immediate learning benefits by taking a mathematical perspective on the various journeys to radicalisation.

2 Modelling journeys through behavioural states

Consider a number of **behavioral states** through which people may transit on their individual journeys. Though the states are common, just like the squares on the snakes and ladders board, each individual's journey may be quite different. Not everybody moves through each state and the residence time within state may be highly individual . The states themselves should be defined as a partition of some multidimensional feature space. That space certainly contains dimensions such as levels of identification with the "cause", level of passive support, preparedness to act and plan. These might be observable in terms of the individuals' use of language and their adoption of the group identity, or their compliance, or gift of passive help or financial support to the cause; or their (actual or inferred) active involvement and their interactions with other known activists.

In reality individuals move through the states in a stochastic way, but by assuming large and well mixed populations in each state we may represent these transitions as bulk population dependent fluxes (at some macroscopic level) rather than tracking each individual's biased random walk (Levy flight or other stochastic process).

Let us begin with such a very simple macroscopic method that analyses populations within a small sequence of states, relying on bulk transition rates, resulting in ordinary differential equations.

2.1 A population based approach

Suppose the general population is stratified into just four groups; one totally inactive "reservoir", and three identifiable groups, representing"aware", states.

This model is thus a simple three state system (a three square snakes and ladders board).

We have an external population, who are inactive as far as the whole idealogy or terrorist activity is concerned. Such individuals are effectively unreachable, uninterested or disabled (even in prison for example) members of the general population. They represent the vast majority of the public.

Then there is a group, G_0, who might become radicalised or interested, but who are currently uninvolved: they are future "possibles". We denote this time dependent population density by $x_0(t)$ (in some suitable units; number of people per km^2 for example). G_0 might constitute a cultural or ethnic subgroup of the general population who are aware and sensitive to the relevant issues and themes. Next there is a group, G_1, of radicalised individuals: those who are committed to the ideology and to the identity or the cause, who might offer passive support or succour, but who are not active as terrorists (in operations or conspiracies). We denote their population density by $x_1(t)$ (in the same units as before). Finally there is a group, G_2, of radicalised and active individuals who are actively plotting attacks on suitable targets. We denote their population density by $x_2(t)$ (again in same units as before).

In this simple model, we shall assume that new individuals are supplied into G_0 at a rate b, say, from the external population. This may be due to either background information arousing their interest, or by children coming of age (or equivalent). For the moment we shall assume that b is a constant (though it may well be dependent upon any news avalanches following the current activities or arrests of individuals within G_2, say).

We will assume that interactions between individuals from G_0 and G_1 cause those from G_0 to be converted, and drawn into G_1. Similarly we will assume that interactions between individuals of G_1 and G_2 cause those from G_1 to be converted, and drawn into G_2.

Finally we will assume that the security services are putting in effort to identify individuals of all three groups and by capture, education, or whatever means will remove them (and return them to the external population: "game over!"). This "identify and remove" process for G_1s and G_2s will be assumed to happen at a rate of a_1 and a_2 per individual.

Note that achieving these removal rates may incur defferent costs. So we write the the sum cost of the security effort in the form

$$C = \pi_1 a_1 + \pi_2 a_2,$$

where the π's are given positive constants. Clearly it is reasonable to assume that $\pi_2 >> \pi_1 > 0$.

Hence if it were affordable we might tolerate G_1s and put enough effort into the removal of G_2s so that the net threat is tolerable ($x_2 <<$ tolerance). On the other hand if a_2 is expensive - tracking lots of possibles and with

expensive interventions - perhaps one could invest heavily into making a_1 large, and depressing x_1, so that G_2s never appear. We will think of the current threat as proportional to $x_2(t)$, the density of actively terrorists.

We have the equations and analysis given in Appendix 1. There are two possible rest-points : a "no-threat" equilibrium (where $x_2 = 0$) and and an "active threat" equilibrium (where $x_2 > 0$). The second exists if and only if $b\gamma_2 > a_1 a_2$ (see Appendix 1).

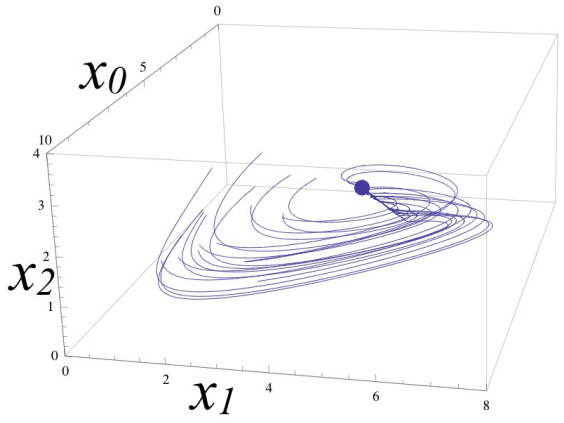

Figure 2. Orbits spiral in to the stable active threat rest point: here $(b, \gamma_1, \gamma_2, a_1, a_2) = (1, 1/10, 1/20, 1/5, 1/5)$ and $(x_0, x_1, x_2) = (5/2, 4/1)$.

In figure 2. we show some orbits approaching the stable "active threat" equilibrium. Notice how these orbits spiral around as the population perturbations (in the lower states) work their way up through the system. Of course a larger model with many more states (more squares on the S&L board) would behave similarly: but we could not visualize it.

Now consider the ability to set or control the capture rates. In the (a_1, a_2) plane the contours of the active , radicaliased population level, $x_2^*(a_1, a_2)$, are the iso-threat contours. Thus one should deploy resources so as to minimise this function under the constraint where C is given. This constrained optimisation problem has an exact solution. The situation is depicted in Figure 3.

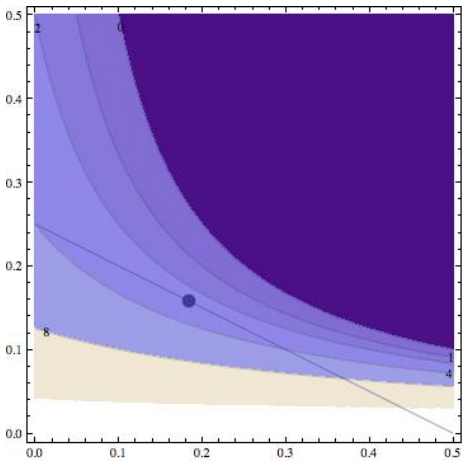

Figure 3. Iso-threat contours for $x_2^*(a_1, a_2)$ (=0,1,2,4,8), the linear constraint and the point of optimal effort.

The central point is that if we settle on the nature of the "snakes and ladders" board, at a macros scale we will obtain a multi state population model, with risk represented by the top, or the top few, populations. Individuals migrate within the whole population in a wide variety of ways. A few of the model rate terms will represent security actions (the snakes rather than ladders), trading individuals downwards, and we should be aware that the distribution of such effort can be optimized. Objects of central concern are the distribution of transit times (from square one to square one hundred) and how these are affected by imposed security controls; and hence how a population bulge entering the board (radicalized by some external or local catalysts) will propagate.

2.2 Coupling-driven instabilities

What happens when there are multiple S&L boards, perhaps representing the state populations within different countries or geographical regions? There may be interactions taking place across these, as individuals communicate, or visit, with their counterparts. In effect this introduces stratified coupling between systems at some of the (higher, more active) state levels. Even when the individual systems are identical and possess very stable equilibria (when considered in insolation), it is not a given that the coupling will merely retain such a global stable uniform equilibrium. Instead the emergence of coupling-driven instability (Turing instability) may mean that the very effects of coupling may be destablelising for otherwise stable situations (especially when this occurs through the higher states).

For example, at its simplest we might consider a 2-group situation of radical activists and foot-soldiers (as in [11]) (denoted by $x_1(t)$ and $x_2(t)$). Suppose that we impose some simple dynamics, for example the Gierer-Meinhart model [14] given in Appendix 2, which would be very appropriate.

Further suppose we have a number of parallel similar systems, representing populations within different states/regions, indexed by $i, = 1, 2, ..., M$.

In general we will let $z_i = (x_1, x_2, ...)^T$ be the state population vector for the ith (region based) system. Then we have

$$\dot{z}_i = F(z_i).$$

Let z^* denote the single system equilibrium, where $F(z^*) = 0$. We shall assume this is stable as a solution of this (uncoupled) system. This is true for the above two state Gierer-Meinhart example.

Next we introduce some stratified coupling across the systems: so that

$$\dot{z}_i = F(z_i) + D_{ij}(z_j - z_i).$$

Here each D_{ij} is a diagnonal matrix and controls horizontal exchanges/flows between the separate ith and jth regional systems (continuity of population mass demands $D_{ij} = D_{ji}$). It is clear that there is still an equilibrium where $z_1, z_2, ...z_M. = z^*$

Now for some activator-inhibitor systems, such as the two state Gierer-Meinhart example in Appendix 2, if there is commonly stronger coupling at the active level (x_2, above) than the more passive level (x_1, above), then a **Turing Instability** [15] can occur, and the uniform equilibrium (where $z_1, z_2, ... = z^*$) becomes unstable. Hence the system within each region will evolve quite differently, despite (and indeed because of) the coupling. Moreover the security services within each region are thus managing an unstable situation.

So it is important to state clearly that such global destabilization of otherwise stable local situations (through direct coupling interactions at certain behavioral state levels of radicalisation) is an inevitable mathematical consequence for some systems: it is not a additional hypothesis.

2.3 A simulation, individual (agent) based, approach

It may be much more productive to simulate a group of individuals' transitions. Such a model would give information about the distribution of transit times to move across the whole set of states; the highest or final state being one containing individuals actively planning and carrying out attacks. This approach should involve an agent based model (ABM), where agents represent individual activists. It is an individual tracking simulation, where at each moment all individuals may or may transit from state to state. Such models can include memory based effects (long term) and also include the selective action of ladders (available to some and not others) and/or capacity constraints. An ABM approach also allows for the agents to influence each other.

Mathematically, we have a set of M individuals, denoted $Y_i(t)$ for $i = 1, ..., M$, where each variable Y_i maps the ith individual at time t onto one

of the set of states. This Y_i takes one of a discrete set of state values. Transitions take place as a result of an individual current state and possibly its history as well as the current external activity and even the influence (and state location) of certain other agents in play. This is a classical complex system, with each individuals own (isolated dynamics) coupled through transient or persistent network effects, and open to external forcing. So we can also allow for one off or occasional (pulsed) activities from the security services.

A simple model could be in discrete time: let $Y_{i,k} = Y_i(t_k)$ denote the state of the ith individual at the k time period (day, week, month,..), corresponding to time t_k.

Let $\mathbf{Y}_k = (Y_{1,k}, Y_{1,k}, ..., Y_{1,k})^T$ denote the states of the entire population (perhaps with state zero represented not yet interested or active - yet to get on the S&L board). Then the evolution of the ith individual will depend on his or her own history; the current state-locations of others; current and recent external events (at home or abroad or in theaters relevant) denoted e_k in the kth time step; and the current and recent actions of the security services, denoted s_k in the kth time step. We have some stochastic evolution rule of the form

$$Y_{i,k+1} = \mathcal{F}(Y_{i,k}, Y_{i,k-1}, ..;\ \mathbf{Y}_k;\ e_k, e_{k-1}, ...;\ s_k, s_{k-1}, ... \mid \lambda_i).$$

For example, given all of the arguments (and history) such information may fix the probability that the individual makes the transition from its current state (Y_k) to any future state. And thus, biased though these possible transitions may be, the individual makes a stochastic state to state transition. Since memory effects are allowed here this is not a simple Markov chain; and we also allow for coupling between individuals, and well as different forms of extrenailties. Here λ_i denotes some fixed parameters employed within the definition of $\mathcal{F}$ that makes individuals distinctive.

Such a model would be amenable to a simulation approach if required. We foresee this as being especially useful where there are strong networking effects. In the population group model, each group is a well mixed sub popluation and groups impact on each other in a uniform (democratically) smoothed manner. But in practice networking and influencing may be highly individual. Hence some individuals will naturally be more isolated while others more influenced. Such a model might be useful in demonstrating what kind of individuals are most likely to propagate through a given state to state model in the fastest time, or with the least chance of churning. Is networking a facilitator or handicap in the individual's personal journey? The evidence from cases such as Nicky Reilly's (the Exeter bomber who grew up with Asperger's Syndrome, which his mother says left him with a mental age of ten and a tendency to develop passing obsessions), indicate that those who are of low self esteem, and are isolated with few or no friends

or community relationships, can be identified, fast-tracked and groomed in a matter of weeks.

3 Observable communications

The digital society within cyber space is boundary-less and is enabled by email, messaging, telephony, online broadcasts (media centers etc), blogs, social media (Facebook, Myspace, Bebo, Twitter) and file sharing (Youtube). It has given rise to 24/7 subjective news and comment, citizen journalism and public scrutiny. Any activity almost anywhere is available for almost anybody: and the rise of the single issue protest or grouping is supported by an empowered and assertive online vox pop.The result is that communication data of very many types is vast and available. There is a growing interest in analyzing this data resource. This data should be used to both inform and calibrate or think about the framework of radicalisation.

For example the time constants involved within a communication system where catalytic events may take place (such as civilian causalities or deaths in a operational theatre) that trigger reaction elsewhere (individuals taking steps to become more involved, or more politicized at least). The alternative also occurs: the home populations' opinions may become hardened by mounting exposure to military causalities over the long term.

3.1 Online activity

It is possible to see which themes and topics people within the UK are searching on Google, or blogging or tweeting about, over time. This can be juxtaposed with government statistical series on catalytic events. In Figure 4. we show some such histories. For the time series shown "p" denotes UK public activity on the internet, while "s" denotes sources statistics (hard data).

The Afghan graphs within these profiles are revealing. There appears to be relatively little public interest in the Afghan War until the latter part of 2009. There may well be some small correlation with UK Forces casualties, linked to specific events: the December peak is probably linked to the crash of the Nimrod where all the crew died, which is still the largest single loss of life in the campaign. It is as if the media impact of the war suddenly gained traction.

The Iraq graphs are also interesting as it appears the public interest in the subject simply waned after 2003. There is the odd correlation with some major bits of news on casualties, but the overall trend on searching is steadily downwards, as if people realised there was little that could be done, so shut it out, as a classic way of dealing with difficult situations.

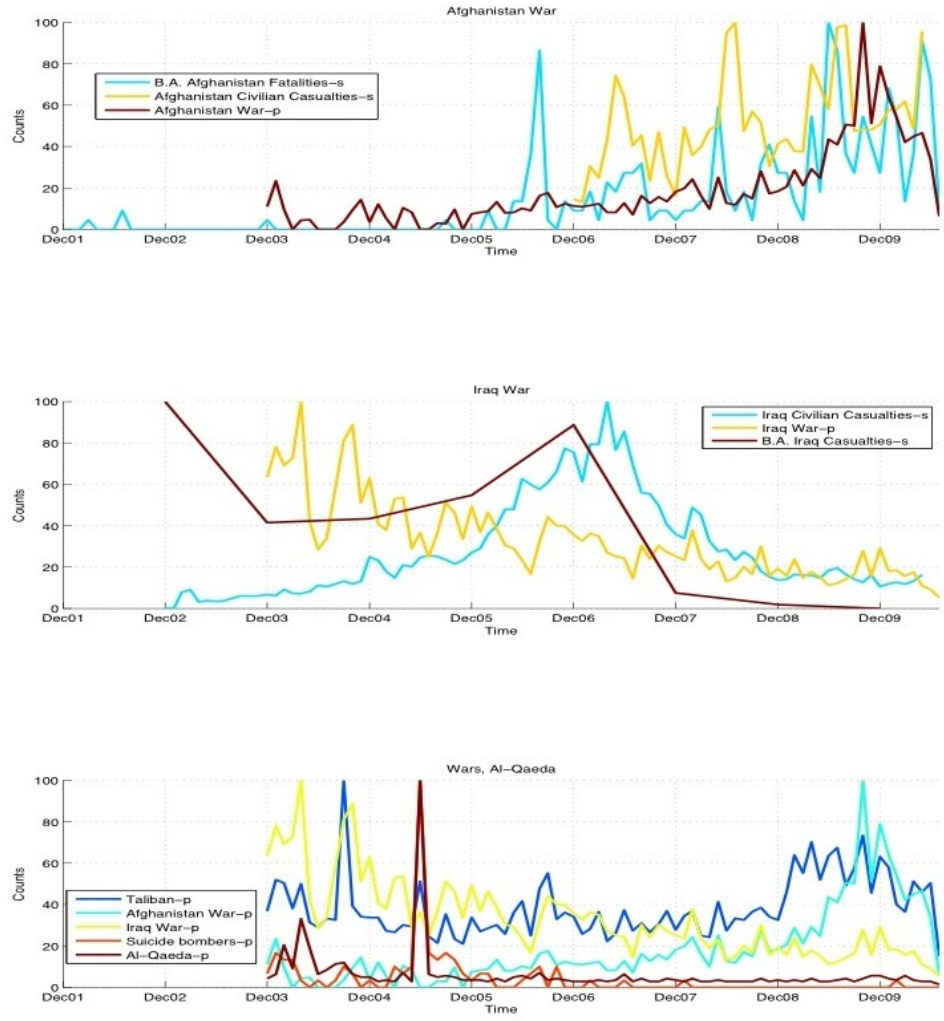

Figure 4. Time series for possible catalytic statistics (s) and UK public internet searches (p).

3.2 Peer to peer communications

Networks for communication are of direct interest in deciding who is influencing and who is being influenced. There has been a growth of mathematical interest in evolving networks, considering the evolving nature of time stamped communications. The time element introduces asymmetry in the way that messages are propagated. A single contact/conversation is clearly symmetric (a phone call for example), each party may brief the other: but a time sequence of such contacts is not. So some individuals may be powerful at getting messages passed around to others whereas other

individuals may be very efficient at hearing everything. The mathematics involves consideration of a sequence of communication networks, and the definition of matrix valued functionals representing the **communicability** of the network; measuring the ability of any specific person to percolate messages, fairly immediately or over time, to any other person [16, 17, 18], and especially [19, 20].

This has opened up a number of exciting possibilities. For example if we have a model for such a network, and we know what has happened today, then we can extrapolate (predict) the evolving network forwards, calculate a prediction of the communicability, and hence make interventions today (injecting information, intervening with certain individuals, or denying contacts) that are optimal.

Consider the case where we have a time stepping sequence of communication networks, labelled by $k = 1, ...K$, , over N vertices (here the vertices are people, and the edges represent two way communications). Each network is represented by a binary symmetric adjacency matrix, A_k (the ijth element is one if node i is in contact with node j, and is zero otherwise). Now, suppose that we wish to quantify the propensity for node i to communicate, or interact with node j, over time. A communication may take place via third parties at different time steps. Although individual networks are symmetric, the composite effect over the time sequence is not. Using our matrix setting, and letting I denote the $N \times N$ identity matrix, these arguments lead us to the calculations of a **Communicability Matrix**, denoted by $\mathcal{Q}$, whose i, j element counts the discounted total number of walks of all possible lengths connecting vertex i to vertex j over the K time steps. We have

$$\mathcal{Q} = (I - aA_0)^{-1} . (I - aA_1)^{-1} . (I - \ aA_2)^{-1} .... (I - aA_K)^{-1} \qquad (1)$$

where the constant a, discounting longer walks, must be chosen small enough. We can use $\mathcal{Q}$ as an overall summary of the ability of the network to achieve pairwise communications over time. The ith row and column sums of $\mathcal{Q}$ measure how effective the ith node is in broadcasting messages to, or receiving messages from all other vertices, respectively. For this reason we will refer to the ith row sum, and the ith column sum, as the source communicability and the sink communicability (of the ith node) respectively.

As an example consider the Enron email data set [21] as an evolving directed network with daily $N \times N$ adjacency matrices, $\{A_k\}_{k=1}^{K}$, where the number of consecutive days is $K = 1138$ and the number of staff is $N = 151$. Day 1 is 11/5/99, Day 1138 is 21/6/02. This data was made public as evidence of possible collusion between named individuals, and used in the US courts. Specifically we use the data on all available emails made from subjects to other subjects (including all multiple recipients designated by "to", "cc", and "bcc" equally) to derive the daily adjacency matrices. Many

of these are empty. We depict the number of emails per day in Figure 5. It is a very sparse sequence at both the begininng and the end.

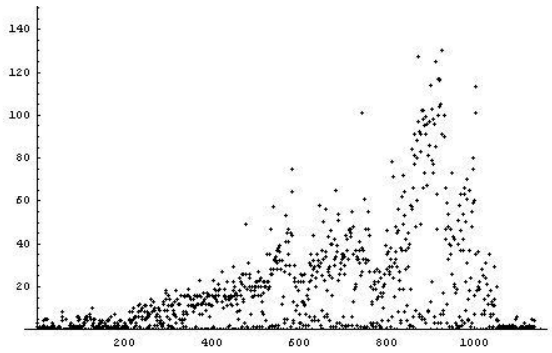

Figure 5. Number of all emails (to all contacts) versus day in the sample.

Next in Figure 6 we show $\mathcal{Q}$ on a (near) log scale below.

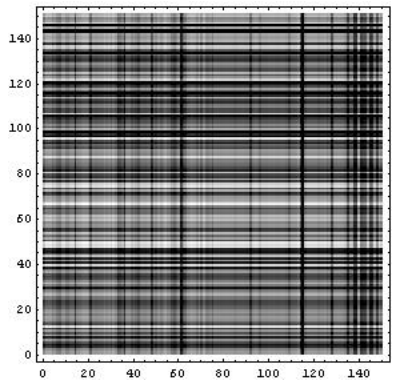

Figure 6. $\mathcal{Q}$, plotting $\ln(1 + \mathcal{Q}_{ij})$.

The source-communicability for each vertex is defined to be the corresponding row sum of the communicabilities in $\mathcal{Q}$, reflecting the ability of the vertex to propagate messages, by all possible discounted walks, to all of the possible recipients. The sink-communicability for each vertex is defined to be the corresponding column sum of the communicabilities in $\mathcal{Q}$, reflecting the ability of the vertex to receive messages by all possible discounted walks, from all of the possible senders. This analysis clearly identifies (named) individuals who were able to influence many others and individuals who were potentially influenced by others.

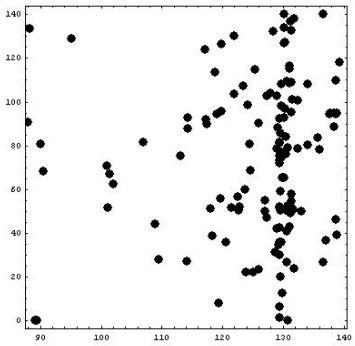

Figure 7. Source-communicability (rows sums of $\mathcal{Q}$) versus sink-communicability (column sums): $a = 0.2$, using natural log scales.

This type of analysis would be amenable to very large networks, if necessary. The linear algebra approach (resolvents for vast sparse matrices) can be adopted using efficient numerical algorithms. Besides the email networks here we have applied it to mobile telephone and other types of messaging data. In our most recent work we have shown how to estimate $\mathcal{Q}$ going forwards, conditioned by some (limited) current data, by exploiting a conditional model for the network's evolution. This would make real time intervention a possibility, based on the conditional predictions of who *will* be playing which type of role. This a a field that is likely to expand rapidly in the next few years.

4 Conclusions

The problem space of radicalisation is certainly one that lends itself to mathematical modelling, at least of many facets, and we have shown that we can gain some practical insights from the application of mathematics to the problem domain. This includes population modelling for estimating levels of activity and relationships between sub populations, radicalisation journey transit times, and the impact of security controls; and ABM simulations for the inclusion of realistic networking and stochastic elements. We have pointed out how parallel and coupled dynamics, representing different regional communities, can and will naturally lead to destabilisation effects within each location. We have illustrated how the availability of data from digital fora and networks may give us insight at both the macroscopic level (in terms of public interest) and the microscopic level (the role of individuals within peer to peer communication networks).

Our use of the snakes and ladders conceptual model attempts to simplify what is in fact a very complex process based upon a range of socio-cultural factors. Presenting such a concept to professional CT audiences has resulted in a very positive response, precisely because it is focussed on the individual's

journey of radicalisation, and integrates across any and all of the complex and interacting enablers. In short it is accessible to all.

This encourages us to define first order effects that create ladders: this being the most significant risk to people who have embarked upon a journey, with the ladders accelerating them towards becoming actively involved in either planning or conducting a terrorist event. If there were no external or events-based ladders, some people would still make the journey though. Perhaps within a group context (being led by a powerful leader) who creates ladders by his arguments and use of a distorted view of religious teachings (his interpretation of the broader ideology). So the mechanisms (and the parameters involved) that create ladders need to be broken down a little more to address how to make snakes. We can then decide if those parameters are combining or fragmenting effects, and ask what drives things together and what creates fragmentation?

Where we believe mathematical modelling may really be able to help most is in looking at the ways that events modulate behaviour and shape subsequent behaviour, via the creation of ladders for people on a journey or a desire to become more actively involved in a social movement, being prepared to sacrifice their own personal identity for a wider cause. This gives us a wider range of parameters that either act to combine to create longer ladders or are pressures that shorten ladders or even create snakes.

Our research here has looked at some of these "event" drivers correlating activity on the WWW associated with specific events. This gives us an indication that there are relationships out there in cyber space that might form the basis of more detailed models.

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We would like to thank our colleagues Dr Reason Machete and Neslihan Yildirim for their help in obtaining the figures shown in section 3, from the web and other public resources.

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Appendix 1: A population model

Using the notation introduced within section 2.1 we have:

$$\dot{x}_0 = b - \gamma_1 x_0 x_1,$$

$$\dot{x}_1 = \gamma_1 x_0 x_1 - \gamma_2 x_1 x_2 - a_1 x_1,$$

$$\dot{x}_2 = \gamma_2 x_1 x_2 - a_2 x_2.$$

Here $\dot{x}_i$ denotes the time derivative dx_i/dt. There are two possible rest point: a "no-threat" equilibrium

$$(x_0^*, x_1^*, x_2^*) = (a_1/\gamma_1, b/a_1, 0),$$

and an "active threat" equilibrium

$$(x_0^*, x_1^*, x_2^*) = (b\gamma_2/a_2\gamma_1, a_2/\gamma_2, -a_1/\gamma_2 + b/a_2).$$

The second exists if and only if $b\gamma_2 > a_1 a_2$. Indeed for $b\gamma_2 < a_1 a_2$ the "no-threat" equilibrium is a stable attracting rest point for the system. At $b\gamma_2 = a_1 a_2$ it suffers a loss of stability, and the "active threat" equilibrium bifurcates away, and continues as a stable rest point, with threat $x_2^*(a_1, a_2) = -a_1/\gamma_2 + b/a_2$, as the product $a_1 a_2$ increases.

Now consider the ability to set or control the capture rates. In the (a_1, a_2) plane the contours of the active , radicalised population level, $x_2^*(a_1, a_2)$, are the iso-threat contours. Thus one should deploy resources so as to minimise this function under the constraint where C is given.

This constrained optimisation problem has an exact solution:

$$a_1^* = \frac{C}{\pi_1} - \sqrt{b\gamma_2}\sqrt{\frac{\pi_2}{\pi_1}}, \quad a_2^* = \sqrt{b\gamma_2}\sqrt{\frac{\pi_1}{\pi_2}},$$

where the threat level is

$$x_2^* = 2\sqrt{\frac{b}{\gamma_2}}\sqrt{\frac{\pi_2}{\pi_1}} - \frac{C}{\pi_1\gamma_2}.$$

We could extend this toy model further. Suppose that activity involving G_2s causes increased recruitment into G_0 at a rate $(b + b'x_2)$; and/or

increased radicalisation causing a fast track (ladder) for individuals in G_0 directly into G_2, at a rate $\mu x_0 x_2$. Then the equations become:

$$\dot{x}_0 = (b + b'x_2) - \gamma_1 x_0 x_1 - \mu x_0 x_2,$$

$$\dot{x}_1 = \gamma_1 x_0 x_1 - \gamma_2 x_1 x_2 - a_1 x_1,$$

$$\dot{x}_2 = \gamma_2 x_1 x_2 - a_2 x_2 + \mu x_0 x_2.$$

The point is that if we settle on the nature of the "snakes and ladders" board, we will obtain a multi state population model, with risk represented by the top, or the top few, populations. A few of the model rate terms represent security actions (the snakes rather than ladders), trading individuals downwards, and we should be aware that the distribution of such effort can be optimised.

Appendix 2: The Gierer-Meinhart model

The Gierer-Meinhart [14]) is one for which Turing (coupling) instabilities occur, when copies are coupled horizontally.

$$\dot{x}_1 = p - x_1 x_2^2 - a x_1,$$

$$\dot{x}_2 = q + x_1 x_2^2 - x_2.$$